Nash equilibrium approach to ML estimation with application to voting models

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Brief description of my work

I propose a new methodology for ML estimation of multinomial choice models. It will be used to re-estimate the probabilistic voting model for several countries. In particular, one can analyze survey data to tell

- How much effect do policy programs of political parties have on voters?
- What do political parties maximize?

Brief description of my work

Consider estimating an econometric model of discrete choice from survey data, such as voting behavior: What party to vote for, depending on the policy positions of the parties, and voter characteristics such as gender, income, etc.?

Existing methodology What values of the model parameters best explain the observed survey response?

Proposed methodology What values of the model parameters best explain the observed survey response AND the policy positions of the parties, given our assumptions on

1. The objective functions of the parties.
2. The information available to the parties.

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Some related works

- Clinton and Meirowitz (2003)
- The quantal response equilirium: McKelvey and Palfrey (1995).

Formal models of political competiton and the empirical puzzle

1. Hotelling (1929), Downs (1957), and Black (1958) — median voter policy coincidence.
2. More than one dimension — equilibrium does not exist in deterministic models. McKelvey (1976, 1979), Plott (1968), McKelvey and Schofield (1987).
3. Policy platforms in real elections do, in fact, diverge.

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Probabilistic voting models

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- Lindbeck and Weibull (1987,1993), Coughlin (1992), Banks and Duggan (2005) — more on MVT.
- Lin, Enelow, and Dorussen (1999), Schofield (2007) — several players.
- McKelvey and Patty (2006) — quantal response equilibrium.
- Patty (2005, 2007) — voteshare maximizers vs. probability of win maximizers.
- Zakharov (2009) — generalized VN-M utility.

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- Adams, Dow, and Merrill (2006), Adams and Merrill (2008), Thurner and Eymann (2000), Plane and Gershtenson (2004), Peress (2005) — abstention in spatial models.
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What are the results?

- Estimation utilizes logit or probit discrete-choice models
- All works show that positions on main policy issues to some degree affect the choice of voters

Models of discrete choice — an overview

Consider a problem of estimating a model of individual choice.

1. A dataset with $i = 1, \ldots, N$ observations, each corresponding to an individual.
2. For each observation, a vector of personal characteristics $x_i \in \mathbf{R}^{M_1}$.
3. and a choice variable $d_i \in \{1, \ldots, J\}$.
4. The utility of individual i choosing an alterantive j is

$$u_{ij} = u(x_i, \alpha_j, \beta, j) + \epsilon_{ij} = \bar{u}_{ij} + \epsilon_{ij}, \quad (1)$$

where $\alpha_j \in \mathbf{R}^{M_2}$ is a vector of choice-specific parameters, and $\beta \in \mathbf{R}^{M_3}$ is a vector of choice-independent parameters.

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We make some assumptions about the distribution of the random variables ϵ_{ij} — usually independence for different values of i. Let $d \in J^N$ denote the choices of all individuals, and $x \in \mathbf{R}^{M_1 N}$ the personal characteristics of all individuals. Our goal is to estimate the values of the parameters $\alpha = (\alpha_j) \in \mathbf{R}^{M_2 J}$, β given our observations (x, d).

An example

Assume that ϵ_{ij} are distributed independently with a Type 1 extreme value distribution:

$$P(\epsilon_{ij} \leq h) = e^{-e^{-h}}. \tag{2}$$

Then, the likelihood of observation i would be

$$P_i = \frac{e^{\bar{u}_{id_i}}}{\sum_{k=1}^{J} e^{\bar{u}_{ik}}}, \tag{3}$$

and of the whole sample —

$$L(x, d, \alpha, \beta) = \prod_{i=1}^{N} P_i. \tag{4}$$

Maximizing L will give us the maximum-likelihood estimates of α and β.

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Assumption 1. There exist K *player agents*. Each player agent k can choose some action y_k from a finite strategy set S_k. Put $S = \times S_k$. Let $y \in S$ denote an action profile for the player agents. For any k and $y \in S$, let y_{-k} be the actions of all player agents other than k.

Assumption 2. The are N *individual* (or *non-player*) agents. The payoff to an individual i choosing an alternative $j \in J$ depends on the actions of the player agents:

$$u_{ij} = u(x_i, \alpha_j, \beta, y, j) + \epsilon_{ij} = \bar{u}_{ij} + \epsilon_{ij}. \tag{5}$$

Individuals observe y before making their choices. d and x are known to the observer.

Assumptions (cont.)

Assumption 3. Every realization of d defines a payoff $U_k(d, y)$ to every player agent k, for every y. The player agents know the true values of the parameters (α, β) and x, but cannot observe ϵ_{ij}s.

Expected payoffs to player agents

Assuming that ϵ_{ij} are independent, the expected payoff of player agent k is

$$\bar{U}_k(x,\alpha,\beta,y) = \sum_{\delta \in J^N} \left(\prod_{i=1}^{N} p_{i\delta_i}(x_i,\alpha,\beta,y) \right) U_k(\delta,y). \qquad (6)$$

where δ runs through all possible choice profiles, and $p_{i\delta_i}$ is the probability that individual i chooses alternative δ_i.

The final assumption

Assumption 4. The observed actions y are a Nash equilibrium in a game with players $1, \ldots, K$, strategy sets S_k, and utilities

$$\tilde{U}_k = \bar{U}_k + \epsilon_k, \tag{7}$$

where ϵ_k are independent random variables. The values ϵ_k are known to all player agents, but not to the observer.

Consider two agent action profiles, y and some y'. Denote by

$$P_j(x, \alpha, \beta, y, y') = P(\tilde{U}_k(x, \alpha, \beta, y) \geq \tilde{U}_k(x, \alpha, \beta, (y'_k, y_{-k})) \quad (8)$$

the probability that agent k choses action y_k over action y'_k, given that all other agents choose y_{-k}.

Suppose that $S = \times S_k$ is a set of action profiles, with $y \in S$.
Suppose that agent k knows that all other agents will choose y_{-k}.
As ϵ_k are independent, the likelihood that he chooses action y_k from S_k is the probability that any pairwise comparison between y_k and any other action $y'_k \in S_k$ is in favor of y_k.
The likelihood of observing $y \in S$ is then

$$L_P(x, \alpha, \beta, y, S) = \prod_{k=1}^{K} \prod_{y'_k \in S_k - \{y_k\}} P_j(x, \alpha, \beta, y, (y'_k, y_{-k})). \quad (9)$$

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The definition of the ML estimator

Definition

Let $0 < \gamma \leq 1$, and S be a set of alternatives for player agents. The *weighted Nash equilibrium maximum likelihood estimator* of (α, β) maximizes the weighted likelihood function

$$L = L_P(x, \alpha, \beta, y, S)^{\gamma} L(x, d, \alpha, \beta)^{1-\gamma}. \tag{10}$$

Example — a probabilistic voting model

Suppose that individuals represent a representative sample of voters. Note that as $K = J$, I will use subscript j to index player agents.

- $x_i \in \mathbf{R}^{M_1}$ — personal characteristics (such as age or income)
- $v_i \in \mathbf{R}^{M_4}$ — individual's preferences with respect to the policies that will be carried out by the winning party in the election
- The choice variable d_i represents the index of the political party that the individual intends to vote for in the upcoming election.

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Example — continued

Let the utility functions of the individuals be given by

$$u_{ij} = a_j + \alpha_j^T x_i + \beta \|v_i - y_j\|^2 + \epsilon_{ij} = \bar{u}_{ij} + \epsilon_{ij}, \qquad (11)$$

where

- a_j is a party-specific constant,
- $\alpha_j \in \mathbf{R}^{M_1}$ is a party-specific vector of parameters,
- β is a parameter, $\| \cdot \|$ is the Eucledian norm,
- $y_j \in \mathbf{R}^{M_4}$ is the policy program of party j,
- ϵ_{ij} is an independent random variable.

Values x_i are usually the socio-economic characteristics of the voter (age, religion, etc).

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$$P(\epsilon_{ij} \leq h) = e^{-e^{-h}}. \tag{12}$$

Then the probability of individual i voting for party j given by

$$p_{ij} = \frac{e^{\bar{u}_{ij}}}{\sum_{h=1}^{J} e^{\bar{u}_{ih}}}. \tag{13}$$

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Assume that the payoff of a political party is equal to the expected number of votes that it will receive in the elections times a constant μ_j. We have

$$U_j(x, \alpha, \beta, y) = \mu_j \sum_{i=1}^{N} p_{ij}. \qquad (14)$$

For each j, this value is a function of individual characteristics x, the parameters α, β, and the policy platforms y.

One can define a game between the J parties, where the strategy of party j is $y_j \in \mathbf{R}^{M_4}$, and the payoff is (6).

Assume that the payoff of a political party is equal to the expected number of votes that it will receive in the elections times a constant μ_j. We have

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Let ϵ_k be distributed as ϵ_{ij}. We have

$$L_P(x,\alpha,\beta,y,S)=\prod_{k=1}^{K}\frac{e^{U_j(x,\alpha,\beta,y)}}{\sum_{y'_j\in S_j-\{y_j\}}e^{U_j(x,\alpha,\beta,(y'_j,y_{-j}))}}. \qquad (15)$$

Let the weighted log likelihood function for this problem be

$$L=w_V L_V+w_P L_P, \qquad (16)$$

where L_V is the likelihood of the observed voting profile, and w_V, w_P are weights.

Example — continued

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Estimating the model: the data

- A dataset for 1996 Israel Knesset elections. Pre-election survey, $N = 922$
- Policy preferences: 2 dimensions, based on 23 questions.
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Estimation: the results

	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 0.8$
Likud	0.7778	0.6135	2.3978
Labor	0.9901	0.6552	1.8475
Mafdal	-0.6270	-1.0018	-0.8998
Modelet	-1.2595	-0.8874	1.7995
Third Way	-2.2916	-2.4721	-0.3101
Shas	-2.0239	-2.5701	-3.0521
β	-1.2075	-1.9050	-3.6245
Log likelihood (voters)	-776.95	-823.0	-1,204.5
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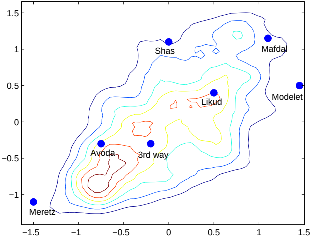
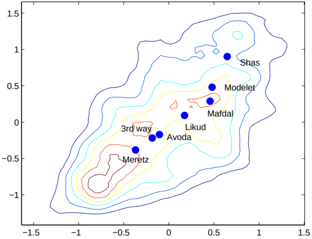
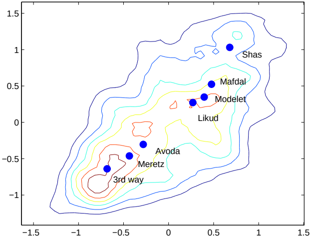


(a) Actual positions



(b) Nash equilibrium, $\gamma = 0$



More estimation: Eurobarometer surveys

- Originally used in Quinn, Martin, and Whitrofd (1998).
- 1977 Netherlands pre-election dataset. $N = 529$.
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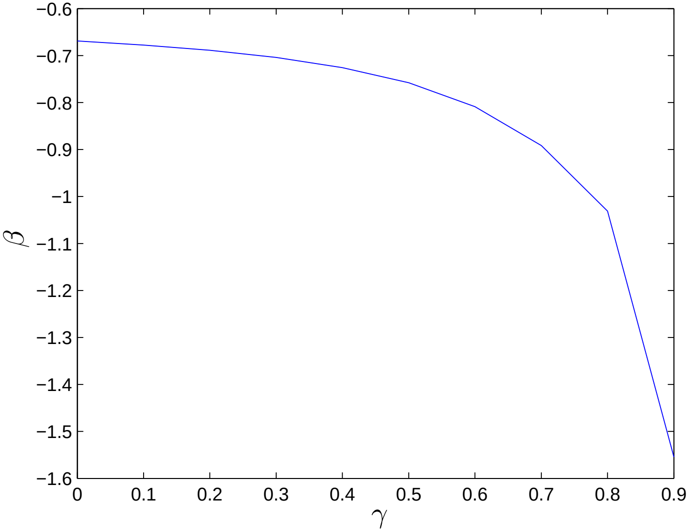

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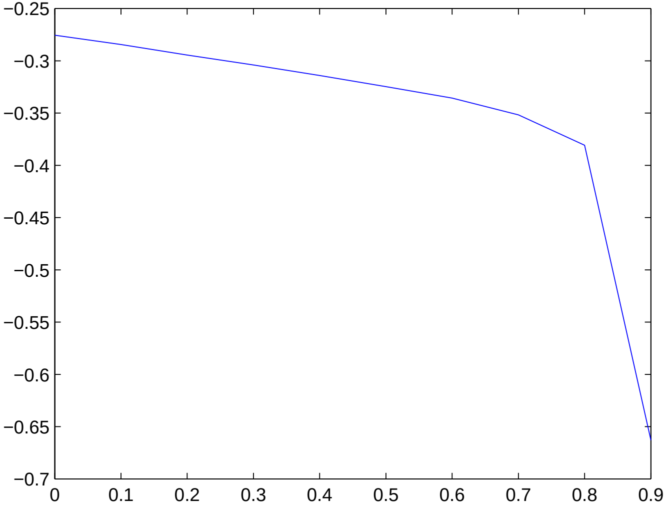

Robustness of estimation

How robust are the estimated parameters with respect to our choice of strategy sets? Other exogenous parameters — μ, γ.

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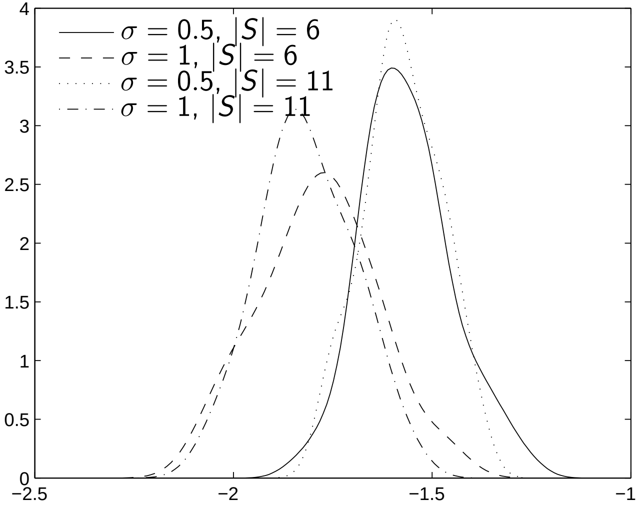

Interpretation

- Larger size of S results in less dispersion of estimated β.
- Larger σ results in larger $|\beta|$.

Size of strategy set

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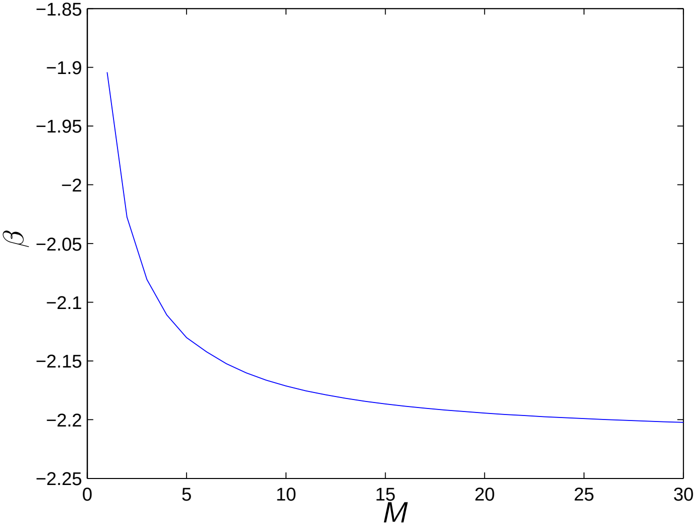
Effect of strategy set size



Asymptotic properties

S are user-defined, but the estimates $\hat{\alpha}$, $\hat{\beta}$ have asymptotic properties. Fix x, d. Suppose that D_k are the k distributions form which the members of the strategy sets S_k are drawn. Define the likelihood of draw y_i as

$$f(y_i|\alpha,\beta) = \prod_{j=1}^{K} P_j(x,\alpha,\beta,y,y_i) \times L^{\frac{1-\gamma}{\gamma}} \quad (17)$$

the probability that the observed actions y are chosen by all player agents, given alternative y_i, times the probability that nonplayer agents choose d. Given M draws we have the likelihood function

$$L(\alpha,\beta|Y) = \prod f(y_i,\alpha,\beta). \quad (18)$$

As we know, the MLE estimator has asymptotic properties, such as consistency $\mathrm{plim}_{M\to\infty}\hat{\beta} = \beta(x)$ and asymptotic normality.

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S are user-defined, but the estimates $\hat{\alpha}$, $\hat{\beta}$ have asymptotic properties.Fix x, d.Suppose that D_k are the k distributions form which the members of the strategy sets S_k are drawn.Define the likelihood of draw y_i as

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A question for further research

What happens to the estimates when both $N \to \infty$ and $N \to \infty$?

Some questions that can be analyzed.

1. Are parties risk-averse?
2. Do parties maximize voteshare or value some specific policy?
3. Are parties forward-looking with respect to coalition formation in cabinet?

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Some problems

There are restrictions on model specification. Consider, for example, the following utility function for political party j:

$$U_j(y) = \eta V_j(y) - (1-\eta)\phi(\|y_j - a_j\|), \tag{19}$$

where $\eta \in [0,1]$, V_j is j's vote share, a_j is j's preferred policy, and $\phi(\cdot)$ is an increasing function.
Then likelihood is maximized at $\eta = 0$ or $\eta = 1$.

The Bayesian approach.

- Computationally intensive
- Bayes ratio test can be used to compare models that are not nested

Thank you