Escalation and Learning

C. Sorokin, E. Winter

2016

Motivation

- International conflicts rarely erupt instantaneously. They almost always result from a process of escalation.
- Potential costs and benefits of engaging in an escalating international conflict are often most uncertain to the parties involved.
- Literature trying to formalize and advance Schelling's early idea of "dancing on the cliff edge" assumed that the sides of the conflict have perfect knowledge of their own possible payoffs.
- We believe that it is an unnecessary simplification:
- The true values of costs and benefits cannot safely be replaced with their expected values — the distributions are different!

Our story

- How players' ability to learn their costs and benefits affects the intensity of escalation?
- We take a comparative statics perspective by allowing both players to learn simultaneously but separately.
- Which may happen, for example, due to availability of better expertise or collection of additional data, improvements in computer technology or just a clearer positions of the third parties.
- So player's own prize is uncertain even to himself, and much more so — to his opponent.
- Thus the players estimate their own prizes using all the information available.
- "Estimation" (or learning) — sampling various signals of the value of the prizes and then using classical Bayesian statistics to update it.

The spoiler

- Our findings indicate that learning about the statistically independent prizes decreases the intensity of escalation and helps in peaceful conflict resolution.
- The worst case happening when parties have to play blindly.
- However, the result partially reverses if the prizes are significantly positive dependent.
- This claim holds even if learning takes place in the middle of the escalating conflict.
- Learning is particularly effective when the stakes are high.

Modeling escalation

- Our ideas may be also applied in many other bilateral conflicts.
- Incentives of parties to reduce tension stem from the understanding that escalation irreversibly lead to a disaster that will leave both parties worse off relative to backing down.
- This property of conflict situations applies to variety of environments and is also relevant for situations of cooperation rather than conflict.

Some other examples

- Sides of a conflict arming themselves to the teeth may loose control on the way the events transpire and find themselves in a very costly war that they didn't expect to occur.
- Party leaders who fight over the leadership as the election day approaches may realise that the battle swings voters' public opinion to the extend that the party is likely to lose the election.
- Two baby birds screaming loud to get their mother's attention while competing over food risk themselves in attracting a dangerous predator.
- Two co-authors procrastinating in making a progress on a paper revision may find that the editor who invited the revision has been replaced and that the paper is no longer under the "revise and resubmit" status.

A long time ago, in a galaxy far, far away ...

- There were two superpowers.
- Each owned enough nuclear warheads to destroy the whole world.
- And they were used to threat to destroy each other in struggle for world dominance.
- John von Neumann was even talking of the strategy of Mutually Assured Destruction (MAD).
- In 1986 Barry Nalebuff published a model that considered an escalating conflict under a probability of mutual destruction.
- In his model, he assumed that "Nuclear war is irrational ... Nuclear deterrence becomes credible only when there exists the possibility for any conventional conflict to escalate out of control".

More references

- Fearon (1994) Domestic Political Audiences and the Escalation of International Disputes.
- Krishna and Morgan (1997) An Analysis of the War of Attrition and the All-Pay Auction.
- Fudenberg and Tirole (1986) A Theory of Exit in Duopoly.
- Maynard Smith, J. & Parker, G. A. (1976) The logic of asymmetric contests.

The model

- Two ex-ante symmetric players.
- $G(t)$ — cdf of the war (disaster) event.
- Both players get 0 utility from war. The player that backed down before war starts gets utility of 1, the other one gets $1 + \tilde{\theta}_i$.
- But player knows only θ_i — his best estimation of the real prize $\tilde{\theta}_i$ (or just his type).
- $F(\theta_1, \theta_2, \tilde{\theta}_1, \tilde{\theta}_2)$ — the "grand" cdf. $F(\theta_1, \theta_2)$ — its marginal on the types, $\tilde{F}(\tilde{\theta}_1, \tilde{\theta}_2)$ — on the prizes.
- $H(\theta_i, \theta_j)$ — the expected value of the prize (for player i) given both types.
- Independent types — $F(\theta_1, \theta_2, \tilde{\theta}_1, \tilde{\theta}_2) = \bar{F}(\theta_1, \tilde{\theta}_1)\bar{F}(\theta_2, \tilde{\theta}_2)$.
- For independent types — $H(\theta_i, \theta_j) = \theta_i$.
- Independent errors — $F(\tilde{\theta}_1, \tilde{\theta}_2 | \theta_1, \theta_2) = F(\tilde{\theta}_1 | \theta_1, \theta_2) F(\tilde{\theta}_2 | \theta_1, \theta_2)$

Features

- While in classical war of attrition model the players' utilities gradually decrease over time, in our model the the cost of escalating arises from the risk of things going out of control in an unknown point of time.
- Parties are ex-ante symmetric.
- Hence we use the symmetric Bayes-Nash equilibrium as our solution concept and do away with equilibrium multiplicity.
- The game is one of incomplete information: the players have only probabilistic beliefs about the other player's benefit from winning the conflict.

Research question

What determines the equilibrium probability of peace?

$$q = 2\int_0^t e^{-\int\limits_0^x \frac{H(y,y)f(y,y)}{\int_y^1 f(y,z)\,dz}dy} \int_x^1 f(y,x)\,dydx.$$

Theorem

The equilibrium probability of peace is independent of the distribution of the disaster event.

So being better or less informed about the disaster does not help, what about having more or less knowledge of the prizes?

Learning

- We need to compare the cases with more and less informed players.
- To do so we assume that players received a number of signals about his prize prior to the game, and that the player's type is just the expected value of the prize given all the signals.
- So the player's are doing Bayesian statistics of all relevant variables as they acquire additional information.
- To maintain symmetry, we assume that both player's are receiving signals and learning, however one player only knows that the other player is learning something, but not what exactly is learned.

Learning-1

- Let $\mathcal{H}_i^k$ denote all the information available to player i at the beginning of the conflict, should he received k signals:

$$\mathcal{H}_i^k = (\theta_i^{(1)}, ..., \theta_i^{(k)}).$$

- Our final goal is to derive a comparative statics result with respect to a number of signals, therefore we assume that the information acquisition process is non-strategic and that player's can't "purchase" additional information as a strategic action.

Learning-2

- The players' prizes and their possible signals have the common prior distribution:

$$F(\tilde{\theta}_1, \tilde{\theta}_2,\ \theta_1^{(1)}, ..., \theta_1^{(k)}, \dots\ \ , \theta_2^{(1)}, ..., \theta_2^{(k)}, ...).$$

- Note that $\tilde{F}(\tilde{\theta}_1, \tilde{\theta}_2)$ is the marginal of $F(\cdot)$ with respect to $\tilde{\theta}_1$ and $\tilde{\theta}_2$.
- We assume that the players' signals are independent conditional on their own true prizes (or, equivalently, that players' signals depend only on their own true prizes and that their estimation errors are independent):

$$F(\theta_1^{(1)}, \dots\ \ , \theta_2^{(1)}, \dots\ \ |\tilde{\theta}_1, \tilde{\theta}_2) = F(\theta_1^{(1)}, \dots\ \ |\tilde{\theta}_1) F(\theta_2^{(1)}, \dots\ \ |\tilde{\theta}_2), \qquad (1)$$

- Note that the signals player i receives need not to be identically and independently distributed.

Learning-3

- Because the true prize is unknown, each player has to estimate it using all the signals available.
- Let θ_i denote the expected prize from player i's perspective given $\mathcal{H}_i$, so that $\theta_i = E(\tilde{\theta}_i|\mathcal{H}_i)$.
- Note that in our setting a player's (true) *prize* and a player's *type* are two different things. The former refers to the realization of a random variable while the latter refers to its conditional expected value.

Lemma

Player i utility given $\mathcal{H}_i^k$ is identical to his utility given θ_i and k, i.e.:

$$E_{\mathcal{H}_j^k}[U_i(t_i, t_j(\cdot))|\mathcal{H}_i^k] \equiv E_{\mathcal{H}_j^k}[U_i(t_i, t_j(\cdot))|\theta_i, k], \quad \text{where } \theta_i = E[\tilde{\theta}_i|\mathcal{H}_i^k].$$

Utilities and FOC

- Denote equilibrium strategy as $t(\theta)$, assume that it is strictly increasing.
- z — player's true signal, x — his "strategic" signal.
- Player's expected utility:

$$(1 - G(t(x)))(1 - F(x|z)) + \int_0^x (1 + H(z, y))(1 - G(t(y)))dF(y|z).$$

- Private values and independent signals:

$$(1 - G(t(x)))(1 - F(x)) + (1 + z) \int_0^x (1 - G(t(y)))dF(y).$$

FOC

By differentiating utilities we obtain

$$-g(t(x))t'(x)(1-F(x))+zf(x)(1-G(t(x))).$$

Or

$$\lambda_G(t(x))=x\lambda_F(x).$$

For the general case:

$$\lambda_G(t(x))=H(x,x)\lambda_F(x|x).$$

Here λ is the hazard rate:

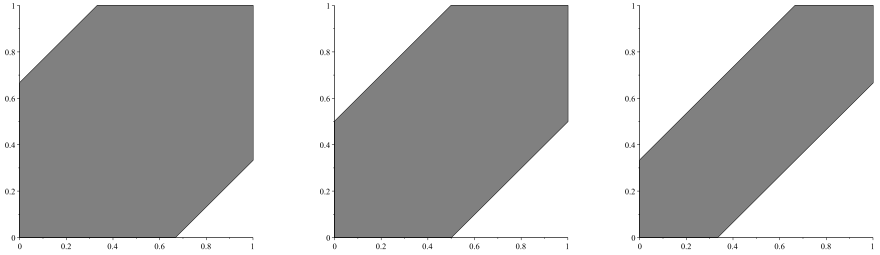
$$\lambda_F(x|z)=\frac{f(x|z)}{1-F(x|z)},\quad \lambda_G(t(x))=\frac{g(t(x))t'(x)}{1-G(t(x))}.$$

SOC

- SOC hold if for all x
$$H(y,x)\lambda_F(x|y)$$
is increasing in y around $y = x$.
- For independent signals SOC hold if player's "valuation" increases with his signal.
- Warning! If components of $F(\theta_1, \theta_2)$ are affiliated, then $\lambda_F(x|y)$ is a decreasing function of y.
- Components of $F(\theta_1, \theta_2)$ are affiliated if $\forall x_1 \leq x_2$ and $\forall y_1 \leq y_2$
$$f(x_1, y_1)f(x_2, y_2) \geq f(x_2, y_1)f(x_1, y_2).$$

Example

As an example of SOC utilization we can take a uniform distribution on a square, "cut" its top-left and bottom-right corners up to a parameter α and then spread the density uniformly within resulting figure.



(a) $\alpha = 1/3$, SOC hold

(b) $\alpha = 1/2$, Border case

(c) $\alpha = 2/3$, SOC fail

Figure: Shape of $f_\alpha(x, y)$ support for different α.

Probability of peace

- By integrating FOC we obtain:

$$1 - G(t(x)) = e^{-\int_0^x H(y,y)\lambda_F(y|y)dy}.$$

- For independent signals and private values:

$$1 - G(t(x)) = e^{-\int_0^x y\lambda_F(y)dy}.$$

- The probability of peace (or peaceful resolution) now equals:

$$q = \int_0^b e^{-\int_0^x y\lambda_F(y)dy} 2(1 - F(x))f(x)dx.$$

Some obvious results

Theorem

The equilibrium probability of peace is independent of $G(t)$.

Theorem

A first-order stochastic increase of F results in lower probability of peace.

Learning

- We need to compare the cases with more and less informed players.
- To do so we assume that players received a number of signals about his prize prior to the game, and that the player's type is just the expected value of the prize given all the signals.
- So the player's are doing Bayesian statistics of all relevant variables as they acquire additional information.
- To maintain symmetry, we assume that both player's are receiving signals and learning, however one player only knows that the other player is learning something, but not what exactly is learned.

Main results

Theorem

For independent prizes any additional information the players acquire increases the probability of peace.

Theorem

The result above holds conditional on any escalation level the game reaches.

- More information — less negative externalities in conflict
- Worst case — players know only the expected value of the prize (no additional information at all).
- We have both upper and lower bounds on probability of peace that depend only on the expected values of the prize.

Common value and interdependent signals

For correlated prizes it may be that:

- If players have scarce information — learning helps.
- If players know a lot — additional information harms by intensifying the competition.

Theorem

If players learn that their prizes are getting closer to each other then they become more aggressive and probability of peace decreases.

Mean-preserving spreads

Let θ and $\hat{\theta}$ be two random variables with CDFs F and $\hat{F}$ respectively. $\hat{\theta}$ is a mean preserving spread of θ if for any convex function $u(\cdot)$ we have

$$\int_0^b u(x)dF(x) \leq \int_0^b u(x)d\hat{F}(x).$$

We will use the following criterion of mean-preserving spread.

Statement

Let θ and $\hat{\theta}$ be two random variables with CDFs F and $\hat{F}$. Suppose that $F^{-1}(\cdot)$ and $(\hat{F})^{-1}(\cdot)$ are well-defined. Then

$$\begin{cases} \int_0^p F^{-1}(y)dy \geq \int_0^p (\hat{F})^{-1}(y)dy, & \forall p \in [0,1), \\ \int_0^1 F^{-1}(y)dy = \int_0^1 (\hat{F})^{-1}(y)dy. \end{cases}$$

if and only if $\theta \leq_{cx} \hat{\theta}$.

Main lemma

Lemma

Let θ and $\hat{\theta}$ be two random variables with CDFs F and $\hat{F}$ respectively and with equal means. Suppose that $\hat{\theta}$ is a mean-preserving spread of θ. Then $q[\hat{F}] \geq q[F]$. In other words, the probability of peace in the symmetric equilibrium increases with mean-preserving spread of players' signals.

Remark

The distribution with minimum spread is obviously a constant:

$$F_{min}(x) = \begin{cases} 0, & \text{if } x \leqslant E_F, \\ 1, & \text{if } x > E_F. \end{cases}$$

$$q[F_{min}] = 1 - \frac{E_F}{2 + E_F}.$$

The distribution with maximal spread is:

$$F_{max}(x) = \begin{cases} 0, & \text{if } x = 0, \\ 1 - \frac{E_F}{b}, & \text{if } x \in (0, b), \\ 1, & \text{if } x = b. \end{cases}$$

$$q[F_{max}] = 1 - \frac{E_F^2}{(2 + b)b}.$$

Observe that it goes to 1 as b goes to $+\infty$*. Also note that* $q[F_{min}] \leq q[F_{max}]$.

- And what do we have in the limit case?
- Now each player's value is E_F, and it is common knowledge.
- No symmetric pure strategies equilibrium now, so lets turn to mixed strategies.
- Assume $\tau(t)$ is a cdf of an equilibrium mixed strategy. Then if first player uses $\tau(t)$ the second one should be indifferent between any two time moments in its support:

$$u_2(t) = (1 - G(t))(1 - \tau(t)) + (1 + E_F) \int_0^t (1 - G(s)) d\tau(s)$$

$$\lambda_G(t) = E_F \frac{\tau'(t)}{1 - \tau(t)} = E_F \lambda_\tau(t), \quad q = \frac{2}{2 + E_F}.$$

Harsanyi's purification theorem holds!

The magnitude of the effect of learning

- This remarks allow us to estimate the magnitude of the effect of learning.
- The distribution of true prizes $\tilde{F}$ to be uniform with support $[0, b]$, so that the expected prize equals $b/2$.
- Recall that the damage from the disaster is 1.
- $q_{uniform}^{peace}$ stands for the probability of peace if players knew their true prizes,
- q_{min}^{peace} is the probability of peace in the worst case — if players know only the expected value of the prize,
- all other cases are somewhere in between.

b	E_f	q_{min}^{peace}	$q_{uniform}^{peace}$	% increase
1	0.5	0.8	0.873	9 %
2	1	0.667	0.792	18%
10	5	0.286	0.533	86%

The learning example–1

- The value of a player's prize $\tilde{\theta}$:

$$\tilde{\theta} \sim \Gamma(\alpha, \beta), \qquad \tilde{f}(x; \alpha, \beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)},$$

- The prizes of both players are dependent:

$$\tilde{F}(x, y) = \tilde{F}(x)\tilde{F}(y)(1 + l(1 - \tilde{F}(x))(1 - \tilde{F}(y))).$$

- Each player receives k signals, each exponentially distributed with parameter equal to player's true prize $\tilde{\theta}_i$
- The signals are independent conditional on player true prizes.

The learning example–2

- The player's posterior distribution of $\tilde{\theta}$ after observing signals $\theta_i^{(1)}, ..., \theta_i^{(k)}$ is Gamma distribution with parameters $\alpha + k$ and $\beta + \sum_{j=1}^{k} \theta_i^{(j)} = \beta + \Theta_i^{(k)}$:

$$f(x|\theta_i^{(1)}, ..., \theta_i^{(k)}) = \left(\beta + \Theta_i^{(k)}\right)^{\alpha+k} \frac{x^{\alpha+k-1} e^{-\beta - \Theta_i^{(k)} x}}{\Gamma(\alpha + k)}.$$

- The expected players prize thus equals:

$$E(\tilde{\theta}_i|\theta_i^{(1)}, ..., \theta_i^{(k)}) = \frac{2 + k}{1 + \Theta_i^{(k)}} = E(\tilde{\theta}_i|\Theta_i^{(k)})$$

The learning example–3

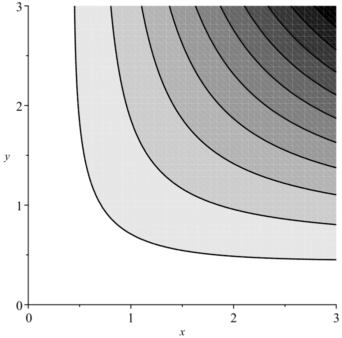
- To run Bayesian game we need to find the distribution of $\theta_i = E(\tilde{\theta}_i | \Theta_i^{(k)})$,
- There is one-to-one correspondence between $\Theta_i^{(k)}$ and $E(\tilde{\theta}_i | \Theta_i^{(k)})$ with

$$\Theta_i^{(k)} = \frac{k + 2 - E(\tilde{\theta}_i | \Theta_i^{(k)})}{E(\tilde{\theta}_i | \Theta_i^{(k)})}.$$

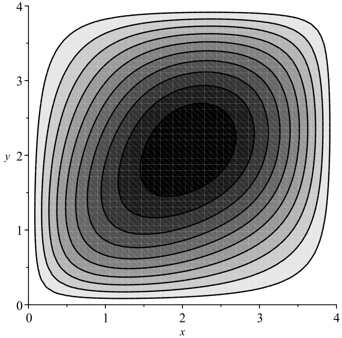
- The sum of k exponentially distributed variables with parameter $\tilde{\theta}_i$ is Gamma distributed with parameters k and $\tilde{\theta}_i$.
- Joint density of $\tilde{\theta}_1$, $\tilde{\theta}_2$, $\Theta_1^{(k)}$, $\Theta_2^{(k)}$.

$$f(\tilde{\theta}_1, \tilde{\theta}_2, \Theta_1^{(k)}, \Theta_2^{(k)}) = \tilde{f}(\tilde{\theta}_1, \tilde{\theta}_2)\gamma(\Theta_1^{(k)}; k, \tilde{\theta}_1)\gamma(\Theta_2^{(k)}; k, \tilde{\theta}_2).$$

The learning example–4a



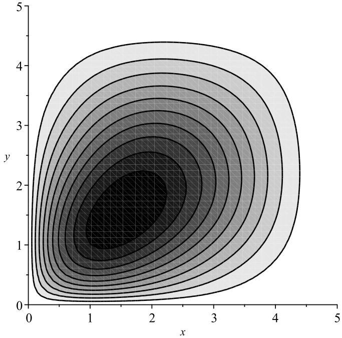
(a) $k = 1$



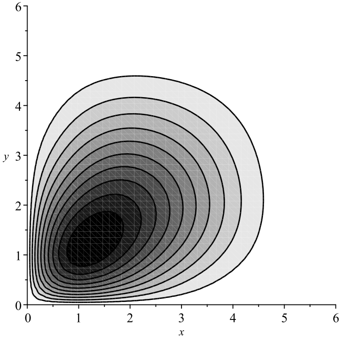
(b) $k = 2$

Figure: Joint density of player prizes $f_k(x, y)$ for different k if $l = 1$. Darker — greater. Note that its support is changing with k.

The learning example–4b



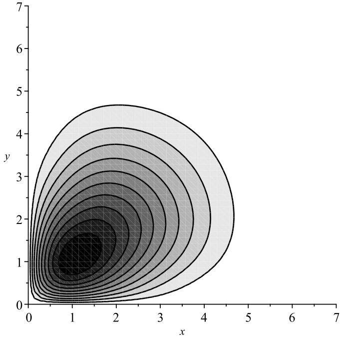
(a) $k = 3$



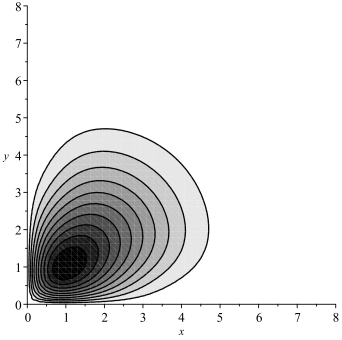
(b) $k = 4$

Figure: Joint density of player prizes $f_k(x, y)$ for different k if $l = 1$. Darker — greater. Note that its support is changing with k.

The learning example–4c



(a) $k = 5$



(b) $k = 6$

Figure: Joint density of player prizes $f_k(x, y)$ for different k if $l = 1$. Darker — greater. Note that its support is changing with k.

The learning example–5

The joint density of player types:

$$f(\theta_1,\theta_2) = \frac{(1+k)^2\,k^2\theta_1\theta_2\,((k-\theta_2+2)(k-\theta_1+2))^{k-1}}{(k+2)^2\,(k+\theta_1+2)^3\,(k+\theta_2+2)^3} *$$

$$* \left(\frac{4l\,(k+2)^4\,((\theta_1+1)\,k+3\,\theta_1+2)\,((\theta_2+1)\,k+3\,\theta_2+2)}{((k+\theta_2+2)\,(k+\theta_1+2))^k} - \right.$$

$$- \frac{2l\,(k+2)^2\,((\theta_1+1)\,k+3\,\theta_1+2)\,(k+\theta_2+2)^3}{((k+\theta_1+2)\,(k+2))^k} -$$

$$- \frac{2l\,(k+2)^2\,((\theta_2+1)\,k+3\,\theta_2+2)\,(k+\theta_1+2)^3}{((k+2)\,(k+\theta_2+2))^k} +$$

$$\left. + \frac{(l+1)\,(k+\theta_2+2)^3\,(k+\theta_1+2)^3}{(k+2)^{2k}} \right).$$

The learning example–6

- But that's not all — we also need to find the function

$$H^{(k)}(x,y) = E(\tilde{\theta}_i | \theta_i^{(k)} = x, \theta_j^{(k)} = y).$$

- Note that this estimate also explicitly depends on k. To derive it we again need to turn to $f(\tilde{\theta}_1, \tilde{\theta}_2, \Theta_1^{(k)}, \Theta_2^{(k)})$ density and calculate the expected value of $\tilde{\theta}_1$ conditional on $\Theta_1^{(k)}$ and $\Theta_2^{(k)}$:

$$E(\tilde{\theta}_1 | \Theta_1^{(k)}, \Theta_2^{(k)}) = \frac{\int\int \tilde{\theta}_1 \tilde{f}(\tilde{\theta}_1, \tilde{\theta}_2)\gamma(\Theta_1^{(k)}; k, \tilde{\theta}_1)\gamma(\Theta_2^{(k)}; k, \tilde{\theta}_2) d\tilde{\theta}_1 d\tilde{\theta}_2}{\int\int \tilde{f}(\tilde{\theta}_1, \tilde{\theta}_2)\gamma(\Theta_1^{(k)}; k, \tilde{\theta}_1)\gamma(\Theta_2^{(k)}; k, \tilde{\theta}_2) d\tilde{\theta}_1 d\tilde{\theta}_2}.$$

- Now we may finish the derivation by substituting

$$\Theta_1^{(k)} = \frac{k+2-x}{x}, \qquad \Theta_2^{(k)} = \frac{k+2-y}{y}$$

into the expression above. Its closed form is provided in the appendix.

The estimate of the prize given both player types:

$$H^{(k)}(x,y) = x * \Big((k+y+2)^3 (k+x+2)^4 (l+1) ((k+y+2)(k+x+2))^k +$$
$$+ (k+2)^2 l \Big(-2(k+x+2)^4 ((y+1)k+3y+2)((k+2)(k+x+2))^k +$$
$$+ (k+2)((x+1)k+4x+2)\left(-2(k+y+2)^3((k+2)(k+y+2))^k +\right.$$
$$\left. + 4\left((k+2)^2\right)^k (k+2)^2((y+1)k+3y+2)\right)\Big)\Big)$$
$$/$$
$$\Big((k+y+2)^4 (k+x+2)^3 (l+1) ((k+y+2)(k+x+2))^k +$$
$$+ (k+2)^2 l \Big(-2(k+x+2)^4 ((y+1)k+3y+2)((k+2)(k+x+2))^k +$$
$$+ ((x+1)k+3x+2)(k+x+2)\left(-2(k+y+2)^3((k+2)(k+y+2))^k +\right.$$
$$\left. + 4\left((k+2)^2\right)^k (k+2)^2((y+1)k+3y+2)\right)\Big)\Big).$$

The learning example–8

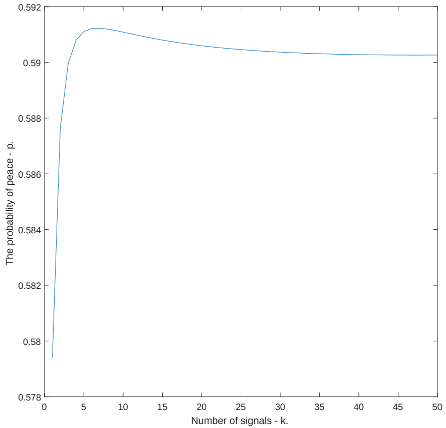


Figure: Probability of peace as a function of number of signals for $l = 1$.

The learning example–9

- So we see that for just one signal the probability of peace equals 0.5794, for 7 signals it equals 0.5912 (maximal value), and for over 30 signals it flattens out at 0.5903.
- Our earlier derivations show that the probability of peace equals 0.5 if the players have to play blind (no signals).
- If players have very scarce information, additional signals play huge role in increasing probability of peace; however, there is a saturation point (eighth signal), when additional signals tell player very little about his own prize but still something about opponent's, thus intensifying the competition and decreasing the probability of peace.
- If we take exponential distributions ($\alpha = 1$) of player true prizes as our starting point, or take parameter l equals just to 0.9 instead of 1, then additional signals only improve the probability of peace.

Suppose that $f(x, y)$ is density of θ and $\hat{f}(x, y)$ is density of $\hat{\theta}$. If the following inequality holds

$$\begin{cases} f(x^1, y^2)\hat{f}(x^2, y^1) \leqslant f(x^1, y^1)\hat{f}(x^2, y^2), \\ f(x^2, y^1)\hat{f}(x^1, y^2) \leqslant f(x^1, y^1)\hat{f}(x^2, y^2) \end{cases}$$

$$\forall x^1 \leq x^2, y^1 \leq y^2, \quad x^1, x^2, y^1, y^2 \in [0, b].$$

then θ is said to be smaller than $\hat{\theta}$ in the multivariate likelihood ratio order (MLRO).

Theorem

A multivariate likelihood ratio dominance shift in joint distribution of players' types may result in higher probability of peace.

Multivariate stochastic dominance-2

This theorem can be proved via a counterexample construction. Take $H(z,x) = z$, $b = 1$, $f(x,y) = 1$ and $\hat{f}(x,y) = 7/2(x^6 + y^6)$.

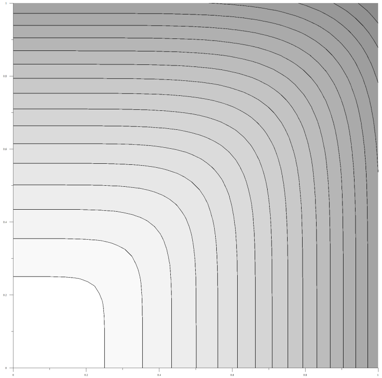


Figure: Plot of $\hat{f}(x,y) = 7/2(x^6 + y^6)$.

What matters?

$$q = 2\int_0^t e^{-\int_0^x \frac{H(y,y)f(y,y)}{\int_y^1 f(y,z)\,dz}dy} \int_x^1 f(y,x)\,dydx.$$

Now recall the SOC: $H(z,x)\lambda_F(x|z)$ should be increasing in z:

$$H(z,x)\lambda_F(x|z) = \frac{H(z,x)f(z,x)}{1-F(x|z)} = \frac{H(z,x)f(z,x)}{\int_x^1 f(y,z)\,dy}$$

So, in order to obtain comparative statics of peace probability with respect to distribution we are interested only in the following:

- the density on the diagonal — $f(x,x)$,
- the density of minimum — $2\int_x^1 f(y,x)\,dy$,
- the SOC: $\frac{H(z,x)f(z,x)}{\int_x^1 f(y,z)\,dy}$ should be increasing in z.

Positive dependence orders

Definition

Let $\theta = (\theta_1, \theta_2)$ and $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ be two pairs of symmetric random variables with joint densities $f(x, y)$ and $\hat{f}(x, y)$ respectively. $(\hat{\theta}_1, \hat{\theta}_2)$ are more *upper diagonally positive dependent* than (θ_1, θ_2) if the following condition holds: $\forall y \in [0, b]$ we have

$$\frac{\hat{f}(y, y)}{\int_y^b \hat{f}(z, y)dz} \geq \frac{f(y, y)}{\int_y^b f(z, y)dz}.$$

Comparative statics in positive dependence

Theorem

Let $\theta = (\theta_1, \theta_2)$ *and* $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ *be player's types. Suppose that SOC hold for both of them and* $\hat{\theta}$ *is more upper diagonally positive dependent than* θ *and* $\min\{\hat{\theta}_1, \hat{\theta}_2\}$ *first-order stochastically dominates* $\min\{\theta_1, \theta_2\}$*, i.e.* $\forall y \in [0, b]$ *we have*

$$\int_y^1 \int_y^1 \hat{f}(x, z)\, dx\, dz \geq \int_y^1 \int_y^1 f(x, z)\, dx\, dz.$$

.

Then probability of peace is greater for θ *than for* $\hat{\theta}$*.*

The most important slide

Thank you!